

- 1) The Broadside array of an 4 -element uniform linear array of element spacing  $d = \lambda/2$  and successive phase shift  $\delta$  . plot the radiation pattern and Find the mainlobe maximum and the sidelobe maxima and all of nulls?

### ① Broadside Antenna Array

Condition  $\rightarrow \delta$  must be equal zero

$$\delta = 0$$

$$\psi = kd \cos \theta + \delta$$

$$\psi = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} \cos \theta + 0$$

$$\psi = \pi \cos \theta$$

To find  $A_{\max}$  assume  $\psi = 0$  at main lobe maximum

$$\therefore \pi \cos \theta = 0$$

$$\theta_{\max} = \cos^{-1}(0) = 90^\circ, 270^\circ$$

Two major lobes are seen because the broadside array is symmetric about  $\theta = \pm \pi/2$

At all of nulls

$$\frac{N\psi}{2} = \pm k\pi \quad ; \text{ where } k = 1, 2, 3, \dots$$

$$\frac{4\pi \cos \theta}{2} = \pm k\pi$$

$$2\pi \cos \theta = \pm k\pi$$

$$\cos \theta_{\text{null}} = \pm \frac{k}{2}$$

$$\text{at 1st null} \rightarrow k=1, \theta = \cos^{-1}(1/2) = 60^\circ$$

$$\theta = \cos^{-1}(-1/2) = 120^\circ$$

$$\text{at 2nd null} \rightarrow k=2, \theta = \cos^{-1}(2/2) = 0^\circ$$

$$\theta = \cos^{-1}(-2/2) = 180^\circ$$

3<sup>rd</sup> null at  $k=3 \rightarrow \cos \theta_{null} = \pm \frac{3}{2}$  X math error  
 max of  $\cos \rightarrow 1$

side lobes

$$\frac{N\psi}{2} = \pm \frac{(2K+1)\pi}{2} \quad ; K = 1, 2, 3, \dots$$

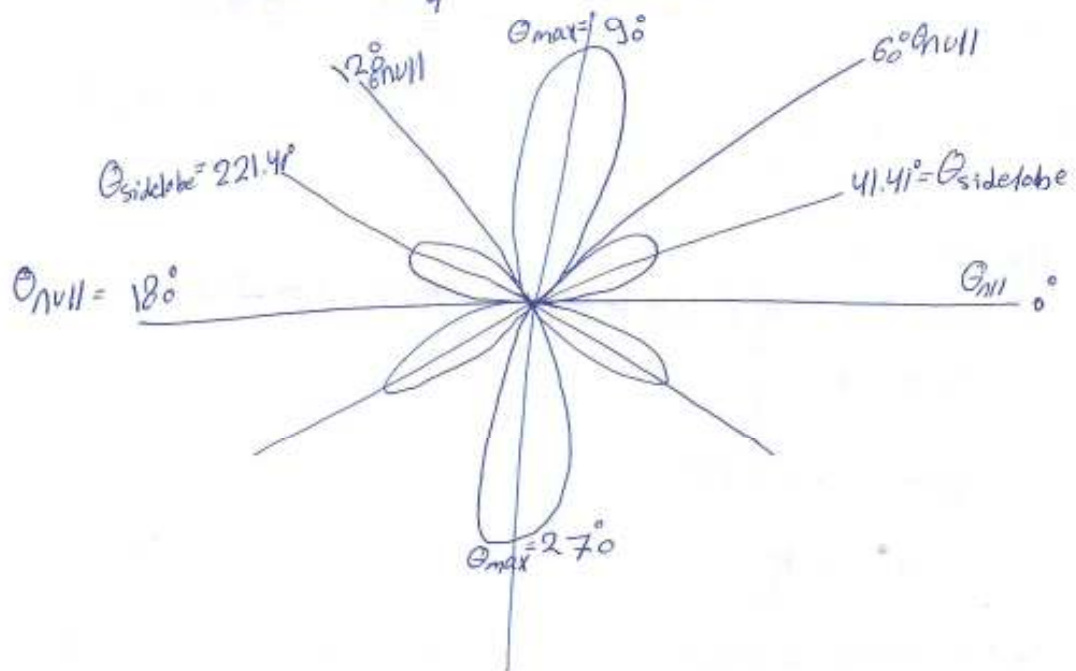
$$\frac{4\pi \cos \theta}{2} = \pm \frac{(2K+1)\pi}{2}$$

$$\cos \theta = \pm \frac{(2K+1)}{4}$$

1<sup>st</sup> sidelobe  $\rightarrow$  at  $k=1 \rightarrow \theta = \cos^{-1}\left(\frac{3}{4}\right) = 41.41^\circ$   
 $\theta = \cos^{-1}\left(-\frac{3}{4}\right) = 221.41^\circ$

2<sup>nd</sup> sidelobe  $\rightarrow$  at  $k=2$

$$\cos \theta = \pm \frac{5}{4} \rightarrow \text{math error} \quad \text{max of } \cos \text{ function} = 1$$



- 2) The End Fire array of an 4 -element uniform linear array of element spacing  $d = \lambda/2$  and successive phase shift  $\delta$ . plot the radiation pattern and Find the mainlobe maximum and the sidelobe maxima and all of nulls?

② End fire Array

$$N = 4, d = \lambda/2$$

Condition  $\delta = -kd$

$$\begin{aligned} \psi &= kd \cos \theta + \delta \\ &= kd \cos \theta - kd \\ &= \frac{2\pi}{\lambda} \cdot \lambda/2 \cos \theta - \frac{2\pi}{\lambda} \cdot \lambda/2 \end{aligned}$$

$$\therefore \psi = \pi \cos \theta - \pi$$

AT main lobe  
find  $\theta_{max}$

$$\psi = \pi \cos \theta - \pi = 0$$

$$\pi \cos \theta = \pi$$

$$\theta = \cos^{-1}(1) = 0^\circ, 180^\circ$$

To find nulls

$$N \frac{\psi}{2} = \pm k\pi$$

$$\frac{4(\pi \cos \theta - \pi)}{2} = \pm k\pi$$

$$\pi \cos \theta - \pi = \pm \frac{k\pi}{2}$$

$$\pi(\cos \theta - 1) = \pm \frac{k\pi}{2}$$

$$\therefore \cos \theta = \pm \frac{(k+1)}{2}$$

At 1<sup>st</sup> null  $\rightarrow k=0 \Rightarrow \theta = \cos^{-1}(1/2) = 60^\circ$

$$\theta = \cos^{-1}(-1/2) = 120^\circ$$

Sidelobes

$$\frac{N\psi}{2} = \pm \frac{(2k+1)\pi}{2}$$

$$2(\pi \cos \theta - \pi) = \pm \frac{(2k+1)\pi}{2}$$

$$\cos \theta - 1 = \pm \frac{(2k+1)}{4}$$

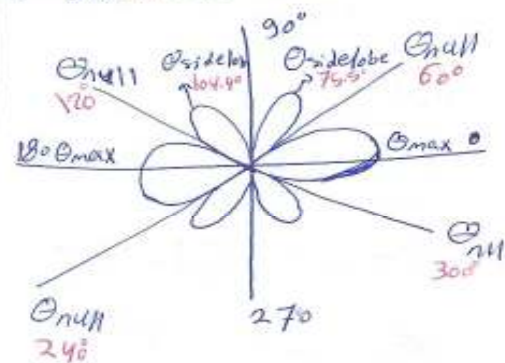
$$\cos \theta = \pm \frac{(2k+1)}{4} + 1$$

$$\cos \theta = \pm \frac{(2k+1)}{4}$$

1<sup>st</sup> sidelobe

$$\text{at } \theta = \pm 75.52^\circ$$

$$2^{\text{nd}} \text{ sidelobe } \theta = \pm 104.47^\circ$$



- 3) The Phased array with a maximum at an angle of  $60^\circ$  from the array line, of an 4 -element uniform linear array of element spacing  $d = \lambda/2$  and successive phase shift  $\delta$ . plot the radiation pattern and Find the mainlobe maximum and the sidelobe maxima and all of nulls?

③ Phased Array

$$N=4, d = \lambda/2, \theta = 60^\circ$$

$$\psi = kd \cos \theta + \delta$$

At mainlobe  $\psi = 0$

$$\frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} \cos 60^\circ + \delta = 0$$

$$\therefore \delta = -\pi \cos 60^\circ$$

$$\delta = -\frac{\pi}{2}$$

$$\therefore \psi = \pi \cos \theta - \frac{\pi}{2}$$

$$0 = \pi \cos \theta - \frac{\pi}{2}$$

$$\cos \theta = \frac{1}{2}, \theta_{\text{mainlobe}} = 60^\circ, 300^\circ$$

Nulls

$$\frac{N\psi}{2} = \pm k\pi$$

$$2(\pi \cos \theta - \frac{\pi}{2}) = \pm k\pi$$

$$2\pi \cos \theta - \pi = \pm k\pi$$

$$2 \cos \theta - 1 = \pm k$$

$$\cos \theta = \frac{\pm(k+1)}{2}$$

at  $k=1$

$$\theta = \cos^{-1}(1) = 0^\circ, 180^\circ$$

at  $k=-1$

$$\theta = \cos^{-1}(-1) = 90^\circ, 270^\circ$$

$$\text{at } k=2, \theta = \cos^{-1}(\frac{1}{2}) = 120^\circ, 240^\circ$$

Sidelobe

$$\frac{N\psi}{2} = \pm \frac{(2k+1)\pi}{2}$$

$$2(\pi \cos \theta - \frac{\pi}{2}) = \pm \frac{(2k+1)\pi}{2}$$

$$2 \cos \theta - 1 = \pm \frac{(2k+1)}{2}$$

$$2 \cos \theta = \pm \frac{2k+1}{2} + 1$$

$$\cos \theta = \frac{\pm(2k+1) + 2}{4} = \pm \frac{(2k+3)}{4}$$

